Exercise 93

A rain drop hitting a lake makes a circular ripple. If the radius, in inches, grows as a function of time in minutes according to $r(t) = 25\sqrt{t+2}$, find the area of the ripple as a function of time. Find the area of the ripple at $t = 2$.

Solution

The area of a circle is

$$A(r) = \pi r^2.$$

But the radius is $r = r(t) = 25\sqrt{t+2}$.

$$\begin{aligned} A(r(t)) &= \pi[r(t)]^2 \\ &= \pi[25^2(t+2)] \\ &= 625\pi(t+2) \end{aligned}$$

The area of the ripple at $t = 2$ is

$$A(r(2)) = 625\pi(2+2) = 2500\pi \approx 7854 \text{ in}^2$$